Coupled Transmission Line/Maxwell-Bloch Simulation Approach for Analysis of Active Mode Locking in Terahertz Quantum Cascade Lasers

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1. Introduction

Active mode locking (AML) via modulation of the injection current or bias is a standard technique employed for the generation of ultrashort pulses in electrically pumped lasers. Quantum cascade lasers (QCLs), as sources of radiation in the mid- and far-infrared portions of the electromagnetic spectrum, have turned out to be exceedingly difficult to actively mode lock, due to the inherently short gain recovery time of these kind of devices [1]. In the mid-infrared, both theoretical and experimental results have shown that this obstacle can be overcome by modulating only a short, electrically isolated section of the QCL cavity [1, 2], which could lead to generation of ultrashort picosecond pulses. For terahertz (THz) QCLs, most recently successful active mode locking of an LO-phonon THz-QCL was reported [3], and pulses as short as 11 ps were detected. Furthermore, in the same work, the importance of correct coupling between the propagating gigahertz (GHz) and terahertz fields was explicitly outlined and the role of the wave-guiding structure in the modulation process emphasized. Here, we present a theoretical model based on the Maxwell-Bloch and the transmission line equations, suitable for investigation of such systems.

2. Theoretical model

We propose a simulation approach for the analysis of electron transport in metal-metal waveguide terahertz QCLs, based on the Maxwell-Bloch and the transmission line equations. A schematic diagram of a typical device, as well as an equivalent circuit representation of a differential section of the waveguide, are presented in Fig. 1a and 1b, respectively.

![Fig. 1. a Schematic diagram of a metal-metal waveguide QCL. A longitudinal current \(i(x,t)\) is assumed to flow along the metallic electrodes, with a corresponding voltage drop \(v(x,t)\) and current density \(J(x,t)\) across the QCL’s active region. b An equivalent circuit representation of a differential section of the waveguide, treated as a parallel plate transmission line with capacitance per unit length \(C'\) and inductance per unit length \(L'\).](image)

In our model, where \(z\) is assumed as the growth direction and \(x\) as the propagation direction, we treat the metal-metal waveguide as a parallel plate transmission line with capacitance and inductance per unit length, \(C'\) and \(L'\), respectively. Sandwiched between the metallic electrodes lies the QCL’s active region (AR), which we model within a density matrix formalism, i.e. Bloch equations, coupled to the optical field via the classical Maxwell’s equations. The transmission line equations are used to resolve the bias voltage \(v(x,t)\) in time and space along the length of the cavity \(l\), as a function of the longitudinal current \(i(x,t)\) and the QCL’s current density \(J(x,t)\). At each time step of our simulation, we interpolate all bias dependent quantities, which enter the density matrix, from a set of precalculated values, obtained with the aid of our Schrödinger-Poisson solver and our ensemble Monte Carlo methods.
Carlo (EMC) simulation code [4]. In this way, we acquire a comprehensive model for the investigation of dynamic electro-optical phenomena, which goes beyond the standard Maxwell-Bloch formalism employed in Refs. [1, 2] and might explain some of the coupling effects suggested in Refs. [3, 5]

3. Results

As a proof of concept, we tested our modelling approach to simulate active mode locking of the device in Ref. [3]. The active mode locking was implemented as a sinusoidal modulation of an externally applied voltage $V_s$, connected via a $Z_s = 50 \Omega$ impedance bonding wire to the left edge of the waveguide. In order to investigate the importance of THz and GHz refractive index matching for successful mode locking [3], we considered three separate simulation scenarios. Setting the THz central frequency’s refractive index at $n_{THz} = 3.6$ and the GHz index at $n_{GHz} = 4.0$, we modulated $V_s$ sinusoidally, i.e. $V_s(t) = V_{sDC}[1 + m \times \sin(2\pi f_{mod} t)]$. When modulation was turned off, Fig. 2a-b, the laser produced a multimode spectrum with equidistant longitudinal modes separated by the free spectral range of $f_{RT} \approx 13.46$ GHz. Alternatively, when the input voltage was modulated at the GHz wave round trip frequency $f_{GHz} \approx 12.49$ GHz, Fig. 2c-d, the simulation produced rich spectral dynamics which we believe are the result of the competition between the propagating THz and GHz waves, i.e. the so called pulling of $f_{RT}$ [5]. Lastly, Fig. 2e-f depicts simulation results when $V_s$ was modulated completely off-resonance. From the resulting spectra as well as beatnote signal, we deduce that the round trip frequency of the THz wave is only weakly perturbed by the voltage modulation and hence at this regime no locking of the round trip frequency is possible.

![Optical field spectrum (arb. u.) and Beatnote (arb. u.)](image)

**Fig. 2.** Results from simulation of the QCL in Ref. [3] with the coupled Maxwell-Bloch/transmission line model. Optical field spectra and the corresponding beatnote for simulation without input voltage modulation, a-b, with voltage modulation at 12.49 GHz, c-d, and with voltage modulation at 13.88 GHz, e-f. For all scenarios the voltage modulation was implemented as $V_s(t) = V_{sDC}[1 + m \times \sin(2\pi f_{mod} t)]$, where $V_{sDC}$ was set to 12 V.

4. References


